A simple SFC model

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1. Brief description

This is a simple SFC model that consists of three sectors: firms, households and banks. Firms undertake investment by using retained profits and loans. A part of firms' profits is distributed to households. Households accumulate savings in the form of deposits. Banks provide firm loans by creating deposits. Banks' profits are distributed to households. In the model, loans are endogenously created when firms receive credit from banks. The model is calibrated using data for the US economy over the period 1960-2010.

The balance sheet matrix and the transactions flow matrix of the model are shown below.

Balance sheet matrix

	Households	Firms	Commercial banks	Total
Deposits	+D		-D	0
Loans		-L	+L	0
Capital		+K		+K
Total (net worth)	+D	$+V_F$	0	+K

Transactions flow matrix

	Households	Firms		Commercial banks		Total
		Current	Capital	Current	Capital	
Consumption	-C	+C				0
Investment		+I	-I			0
Wages	+W	-W				0
Firms' profits	+DP	-TP	+RP			0
Banks' profits	+BP			-BP		0
Interest on deposits	$+int_D D_{-1}$			$-int_D D_{-1}$		0
Interest on loans		$-int_L L_{-1}$		$+int_L L_{-1}$		0
Change in deposits	$-\Delta D$				$+\Delta D$	0
Change in loans			$+\Delta L$		$-\Delta L$	0
Total	0	0	0	0	0	0

2. Model equations

Households:

Wage income of households: $W = s_W Y$ (1)

Capital income of households: $Y_C = DP + BP + int_D D_{-1}$ (2)

Consumption expenditures: $C = c_1 W_{-1} + c_2 Y_{C-1} + c_3 D_{-1}$ (3)

Deposits (identity): $D = D_{-1} + W + Y_C - C$ (4)

Firms:

Output: $Y = C + I$ (5)

Total profits of firms (identity): $TP = Y - W - int_L L_{-1}$ (6)

Retained profits: $RP = s_F TP_{-1}$ (7)

Distributed profits (identity): $DP = TP - RP$ (8)

Investment: $I = g_K K_{-1}$ (9)

Capital stock: $K = K_{-1} + I$ (10)

Loans (identity): $L = L_{-1} + I - RP$ (11)

Banks:

Profits of banks (identity): $BP = int_L L_{-1} - int_D D_{-1}$ (12)

Deposits (redundant identity): $D_{red} = L$ (13)

Auxiliary equations:

Potential output: $Y^* = vK$ (14)

Capacity utilisation: $u = Y / Y^*$ (15)

Growth rate of output: $g_Y = (Y - Y_{-1})/Y_{-1}$ (16)

Leverage ratio: $lev = L / K$ (17)

3. Symbols and values

Symbol	Description	Value/calculation
	Parameters	
c_1	Propensity to consume out of wage income	0.9 [Category B(iii)]
c_2	Propensity to consume out of capital income	0.75 [Category B(iii)]
c_3	Propensity to consume out of deposits	0.473755074 [Category C(ii)]
g_K	Growth rate of capital	US 1960-2010 mean value of g_Y [Category C(ii)]
int_D	Interest rate on deposits	US 1960-2010 mean value of int_D [Category B(i)]
int_L	Interest rate on loans	US 1960-2010 mean value of int_L [Category B(i)]
s_F	Retention rate of firms	0.17863783 [Category C(ii)]
s_W	Wage share	US 1960-2010 mean value of s_W [Category (Bi)]
v	Capital productivity	Calculated using equations (14) and (15) [Category C(i)]
	Endogenous variables	
W	Wage income of households	Calculated from equation (1)
Y_C	Capital income of households	Calculated from equation (2)
C	Consumption expenditures	Calculated from equation (5)
D	Deposits	Calculated from equation (13)
Y	Output	US 1960 value (in trillion 2009 US$)
TP	Total profits of firms	Calculated from equation (6)
RP	Retained profits	Calculated from equation (7)
DP	Distributed profits	Calculated from equation (8)
I	Investment	Calculated from equation (9)
K	Capital stock	US 1960 value (in trillion 2009 US$)
L	Loans	US 1960 value (in trillion 2009 US$)
BP	Profits of banks	Calculated from equation (12)
D_{red}	Deposits (redundant)	Calculated from equation (13)
Y^*	Potential output	Calculated from equation (14)
u	Capacity utilisation	US 1960 value
g_Y	Growth rate of output	US 1960-2010 mean value of g_Y
lev	Leverage ratio	Calculated from equation (17)

Note: For the different categories of parameters, see Appendix B

4. Steps for simulating the model in R

#Open R and create a new R script (File->New file->R script). Save this file as 'Model' (File->Save as).

#Clear the workspace and identify how many time periods (T) you wish your model to run. Since we use US data for the period 1960-2010, we will run the model for 51 periods. (Once you have written the commands, press 'Source'.)

```
rm(list=ls(all=TRUE))
T<-51
```

#Download the excel file that contains the US data for the period 1960-2010 that will be used for the calibration of the model (the data come from FRED and BIS). Save the file as .csv in your desktop and insert it into R using the command below. (Once you have written the command, press 'Source'.)

Data<- read.csv("C:/users/user/Desktop/Data.csv")
#This should be adjusted based on the location of your file; if you have problems in reading the file, you could potentially try to use a command like this one:
Data<- read.csv("C:/users/user/Desktop/Data.csv, dec = "," , sep = ";")

#If you wish to estimate the mean of a variable use a command like this one (type this in Console): mean(Data[,c("g_Y")])

#STEP 1: Identify the endogenous variables of the model (as well as some auxiliary variables). For each of them create a vector that has a length equal to the time periods. (Once you have written the commands, press 'Source'.)

```
#Endogenous variables
W<- vector(length=T)
Y_C<- vector(length=T)
CO<- vector(length=T)
D<- vector(length=T)
Y<- vector(length=T)
TP<- vector(length=T)
RP<- vector(length=T)
DP<- vector(length=T)
I<- vector(length=T)
K<- vector(length=T)
L<- vector(length=T)
BP<- vector(length=T)
D_red<- vector(length=T)
Y_star<- vector(length=T) #auxiliary variable
u<- vector(length=T)  #auxiliary variable
g_Y<- vector(length=T) #auxiliary variable
lev<- vector(length=T) #auxiliary variable
```

#STEP 2: Identify the parameter values based on the information that is available in Section 3. Note that in our baseline scenario we wish our model to be at a steady state whereby economic growth is equal to the mean economic growth in the US in 1960-2010.

```
#Parameters
for (i in 1:T) {
if (i == 1) {
  for (iterations in 1:10){

c_1<-0.9
c_2<-0.75
c_3<-0.473755074#(K[i]/L[i])*((Y[i]/K[i])*(1+g_K)-g_K-(c_1*W[i]/K[i]+c_2*Y_C[i]/K[i]))
g_K<- mean(Data[,c("g_Y")])
int_D<- mean(Data[,c("int_D")])
int_L<- mean(Data[,c("int_L")])
s_F<-0.17863783# (g_K-g_K*(L[i]/K[i]))/(TP[i]/K[i])
s_W<-mean(Data[,c("s_W")])
v<-Y[i]/(K[i]*u[i])
```

#STEP 3: Select the initial values using the data for your economy or the equations of the model (see Section 3).

```
#Initial values
W[i]<-s_W*Y[i]
Y_C[i]<-DP[i]+BP[i]+int_D*(D[i]/(1+g_K))
```

```
CO[i]<-Y[i]-I[i]
D[i]<-L[i]
Y[i]<-Data[1,c("Y")]
TP[i]<-Y[i]-W[i]-int_L*(L[i]/(1+g_K))
RP[i]<-s_F*TP[i]/(1+g_K)
DP[i]<-TP[i]-RP[i]
I[i]<-(g_K/(1+g_K))*K[i]
K[i]<-Data[1,c("K")]
L[i]<-Data[1,c("L")]
BP[i]<-int_L*(L[i]/(1+g_K))-int_D*(D[i]/(1+g_K))
D_red[i]<-L[i]
Y_star[i]<-v*K[i]
u[i]<-Data[1,c("u")]
g_Y[i]<-g_K
lev[i]<-L[i]/K[i]

  }
 }
```

#STEP 4: Write down the equations and run the model. (Once you have written the commands, press 'Source'.)

```
#Equations
else {

for (iterations in 1:10){

#Households
W[i]<-s_W*Y[i]
Y_C[i]<-DP[i]+BP[i]+int_D*D[i-1]
CO[i]<-c_1*W[i-1]+c_2*Y_C[i-1]+c_3*D[i-1]
D[i]<-D[i-1]+W[i]+Y_C[i]-CO[i]

#Firms
Y[i]<-CO[i]+I[i]
TP[i]<-Y[i]-W[i]-int_L*L[i-1]
RP[i]<-s_F*TP[i-1]
DP[i]<-TP[i]-RP[i]
I[i]<-g_K*K[i-1]
K[i]<-K[i-1]+I[i]
L[i]<-L[i-1]+I[i]-RP[i]

#Banks
BP[i]<-int_L*L[i-1]-int_D*D[i-1]
D_red[i]<-L[i]

#Auxiliary equations
Y_star[i]<-v*K[i]
u[i]<-Y[i]/Y_star[i]
g_Y[i]<-(Y[i]-Y[i-1])/Y[i-1]
lev[i]<-L[i]/K[i]

   }
 }
}
```

#STEP 5: Report your results by using tables and graphs. In the graphs that we create here we compare the actual and the simulated data. (Once you have written the commands, press 'Source'.)

```
#Table
matrixname<-paste("Table")
assign (matrixname, (round(cbind(D_red, D, u, g_Y, lev, Y), digits=4)))

#Graphs
plot(Data[1:T,c("lev")], type="l", xlab= "Year", ylab= "Leverage ratio", xaxt="n")
lines(Table[1:T,c("lev")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")

plot(Data[1:T,c("u")], type="l", xlab= "Year", ylab= "Capacity utilisation", xaxt="n")
lines(Table[1:T,c("u")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")

plot(Data[1:T,c("g_Y")], type="l", lty=1, xlab= "Year", ylab= "Growth rate of output", xaxt="n")
lines(Table[1:T,c("g_Y")], type="l", lty=3)
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")

plot(Data[1:T,c("Y")], type="l", lty=1, xlab= "Year", ylab= "Output", xaxt="n")
lines(Table[1:T,c("Y")], type="l", lty=3 )
axis(side=1, at=c(1,11,21,31,41, 51), labels=c("1960","1970","1980", "1990","2000","2010"))
legend("bottomright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

 # Suppose now that we wish to make one of our parameter values (the wage share) endogenous and subject to exogenous shocks based on the data. We thereby allow s_w to take the values from the data.

Replace:
```
s_W<-mean(Data[,c("s_W")])
```

with:
```
s_W<-(Data[,c("s_W")])
```

Also, replace 's_W' in the initial values and the equations with 's_W[i]'.

(Once you have have done the above, press 'Source'.)

#STEP 6: Validate the model. Here we estimate only the autocorrelation for output. (Once you have written the commands, press 'Source'.)

```
#install.packages("mFilter") #this command is necessary if mFilter has not been installed in your computer

library(mFilter)

Y_log<-log((Table[,c("Y")]))
Yactual_log<-log((Data[,c("Y")]))

Y.hp <- hpfilter((Y_log), freq=6.25, drift=TRUE)
actualY.hp <- hpfilter((Yactual_log), freq=6.25, drift=TRUE)
```

```
acfYactual=acf(actualY.hp$cycle, lag.max=20, plot=F)
acfY=acf(Y.hp$cycle,lag.max=20, plot=F)
plot(acfYactual$acf, ylab=" ", xlab="Lag", type="l", lty=1, ylim=c(-0.5,1))
lines(acfY$acf, type="l", lty=3, ylim=c(-0.5,1))
legend("topright", legend=c("Actual", "Simulated"), lty=c(1,3), bty="n")
```

#STEP 7: Re-run the simulations by changing key parameters (we skip this step here).

#STEP 8: Re-run the simulations by changing parameters that correspond to policies/institutional structures.

#First, assume that the wage share is equal to its real value in the US till 1980 and equal to 0.55 thereafter.

Create a vector for s_w:
s_W<- vector(length=T)

Put the command below after ‘**(**i **in** 1:T**) {**‘
if (i<21){s_W[i]<-Data[i,c("s_W")]} else {s_W[i]<-0.55}

Delete the command:
s_W<-(Data[,c("s_W")])

(Once you have have done the above, press ‘Source’.)

#Second, assume that the loan interest rate is equal to its mean value in the US till 1980, and equal to 0.25 thereafter.

Delete the following commands in order to cancel the wage share shock:
s_W<- vector(length=T)
if (i<21){s_W[i]<-Data[i,c("s_W")]} else {s_W[i]<-0.55}

Use again the following command:
s_W<-(Data[,c("s_W")])

Put the command below after ‘**(**i **in** 1:T**) {**‘
if (i<21){int_L<-mean(Data[,c("int_L")])} else {int_L<-0.25}

Delete the command:
int_L<- mean(Data[,c("int_L")])

(Once you have have done the above, press ‘Source’.)

Appendix A: Categories of parameter values

Category	Description
(A)	**Econometrically estimated parameters**
(B)	**Directly calibrated parameters**
(Bi)	Based on data
(Bii)	Based on previous studies
(Biii)	Selected from a reasonable range of values
(C)	**Indirectly calibrated parameters**
(Ci)	Calibrated such that the model matches the data
(Cii)	Calibrated such that the model generates the baseline scenario

Appendix B: Estimating c_3 and s_F for the baseline scenario

In the baseline scenario we wish our economy to be at a steady state whereby economic growth is equal to the mean economic growth in the US in 1960-2010. This implies that the ratios Y/K and L/K should be constant. We calibrate the parameters c_3 and s_F in order to achieve that.

(a) Calibrating c_3 such that Y/K is constant

$$\Delta\left(\frac{Y}{K}\right)=\frac{Y}{K}-\frac{Y_{-1}}{K_{-1}}=\frac{Y}{K}-\frac{Y_{-1}\left(1+g_K\right)}{K}=\frac{\Delta Y-g_K Y_{-1}}{K}=\frac{\Delta Y}{K}-\frac{Y}{K}\frac{g_K}{\left(1+g_K\right)} \qquad \text{(B1)}$$

We have: $\frac{\Delta Y}{K}=\frac{C+I-Y_{-1}}{K}=\left(\frac{C}{K_{-1}}+\frac{I}{K_{-1}}-\frac{Y_{-1}}{K_{-1}}\right)\frac{K_{-1}}{K}=\left(\frac{c_1W_{-1}+c_2Y_{c-1}+c_3D_{-1}}{K_{-1}}+g_K-\frac{Y_{-1}}{K_{-1}}\right)\frac{K_{-1}}{K}$ or

$$\frac{\Delta Y}{K}=\left(\frac{c_1W_{-1}}{K_{-1}}+\frac{c_2Y_{c-1}}{K_{-1}}+\frac{c_3D_{-1}}{K_{-1}}+g_K-\frac{Y_{-1}}{K_{-1}}\right)\frac{K_{-1}}{K} \text{ or}$$

$$\frac{\Delta Y}{K}=\left(\frac{c_1W\left(1+g_K\right)}{K\left(1+g_K\right)}+\frac{c_2Y_c\left(1+g_K\right)}{K\left(1+g_K\right)}+\frac{c_3D\left(1+g_K\right)}{K\left(1+g_K\right)}+g_K-\frac{Y\left(1+g_K\right)}{K\left(1+g_K\right)}\right)\frac{K}{K\left(1+g_K\right)} \text{ or}$$

$$\frac{\Delta Y}{K}=\left(\frac{c_1W}{K}+\frac{c_2Y_c}{K}+\frac{c_3L}{K}+g_K-\frac{Y}{K}\right)\frac{1}{\left(1+g_K\right)} \qquad \text{(B2)}$$

Substituting (B2) into (B1), we get:

$$\Delta\left(\frac{Y}{K}\right)=\left(\frac{c_1W}{K}+\frac{c_2Y_c}{K}+\frac{c_3L}{K}+g_K-\frac{Y}{K}\right)\frac{1}{\left(1+g_K\right)}-\frac{g_KY}{K\left(1+g_K\right)} \qquad \text{(B3)}$$

Since Y/K should be constant at the steady state, we need $\Delta\left(\frac{Y}{K}\right)=0$. Solving (B3) for c_3:

$$c_3=\frac{K}{L}\left(\frac{Y}{K}\left(1+g_K\right)-g_K-\left(\frac{c_1W}{K}+\frac{c_2Y_c}{K}\right)\right)$$

(b) Calibrating s_F such that L/K is constant

$$\Delta\left(\frac{L}{K}\right)=\frac{L}{K}-\frac{L_{-1}}{K_{-1}}=\frac{L}{K}-\frac{L_{-1}\left(1+g_k\right)}{K}=\frac{\Delta L-g_KL_{-1}}{K}=\frac{\Delta L}{K}-\frac{L}{K}\frac{g_K}{\left(1+g_K\right)} \qquad \text{(B4)}$$

We have:

$$\frac{\Delta L}{K}=\frac{I-RP}{K}=\left(\frac{I}{K_{-1}}-\frac{RP}{K_{-1}}\right)\frac{K_{-1}}{K}=\left(g_K-s_F\frac{TP_{-1}}{K_{-1}}\right)\frac{K}{K\left(1+g_K\right)}=\left(g_K-s_F\frac{TP}{K}\right)\frac{1}{\left(1+g_K\right)} \qquad \text{(B5)}$$

Substituting (B5) into (B4), we get:

$$\Delta\left(\frac{L}{K}\right)=\left(g_K-s_F\frac{TP}{K}\right)\frac{1}{\left(1+g_K\right)}-\frac{g_KL}{\left(1+g_K\right)K} \qquad \text{(B6)}$$

Since L/K should be constant at the steady state, we need $\Delta\left(\frac{L}{K}\right)=0$. Solving (B6) for s_F:

$$s_F = \left(g_K - g_K \frac{L}{K}\right) \bigg/ \frac{TP}{K}$$

Appendix C: An alternative investment function

The following non-linear investment function allows you to generate endogenous cycles in the model. According to this investment function, investment is a positive function of capacity utilisation and a negative function of leverage.

```
I[i]<-(2*(mean(Data[,c("g_K")]))/(1+exp(-10*(u[i-1]-0.8)+180*(lev[i-1]-0.12))))*K[i-1]
```